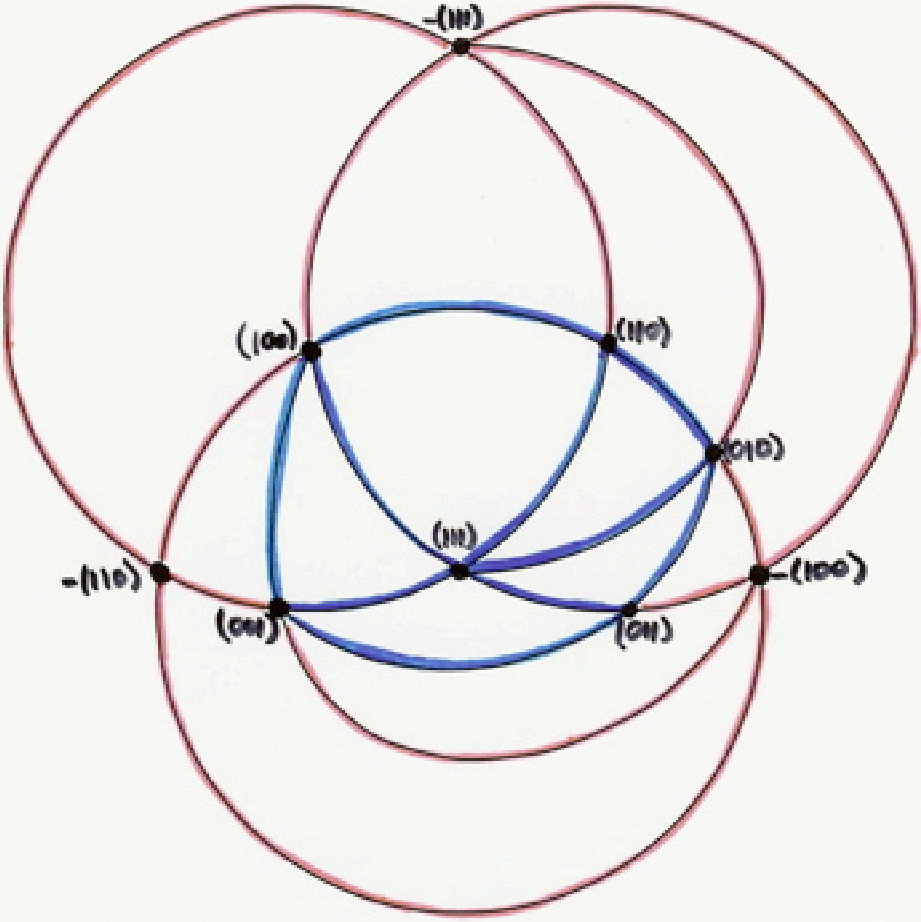
-(111)
(100)
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(111)
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(011)

(2) Nilpotent, Torsion free groups

- $G = G_0 \supset G_1 \supset G_2 \supset \ldots$, $G_{i+1} = [G G_i]$
- G is <u>nilpotent</u> if $\exists s \ni G_s = 0$
- G is <u>torsion free</u> if G_i/G_{i+1} f.g. free abelian

Remark: There exists a finite basis $B = \{b_1, \ldots, b_n\}$ for nilpotent torsion free groups.

Associate Lie algebra L $[a_iG_{i+1}, b_jG_{j+1}] := [ab]G_{i+j+1}$
Lie bracket commutator

Homology of G and L

(Nomizu) Rational homology of G
$\cong$
Rational homology of L

Integral homology of G
not $\cong$
Integral homology of L

Koszul complex gives integral homology for L
rational homology for L and G

Conkel + Porter — results on integral homology of G.

③ $\boxed{\text{Igusa-Orr complex and pictures}}$

Motivation: • From topology

• Milnor $\bar{\mu}$-invariants of links

• K. Orr proved that all $\bar{\mu}$ invariants are in H_3 of the fundamental group of the complement of the link.

• IO constructed complex and proved that H_3 has no torsion for nilpotent free groups (ie free / higher commutat)

Igusa-Orr complex + pictures

- construct a particular $\mathbb{Z}[G]$ resolution of $\mathbb{Z}$.
- freely generated on all subsets of the basis B.
- Recursive, with complete description (i.e. for each subset of B).
- Boundary is Koszul boundary + additional terms.
- Boundary for each subset of B corresponds to a "picture" (i.e. triangulation of sphere, satisfying certain conditions – nec. + suff.).
- commutator is the most efficient "collecting process"
- start with commutators.

$aba^{-1}b^{-1}$

$a^{-1}b^{-1}ab \longleftarrow$ Igusa-Orr pictures

$b^{-1}aba^{-1} \longleftarrow$ Cluster - semi-invariant pictures

5

Monomial groups

Def G <u>monomial</u> if:
- nilpotent
- torsion free
- $\exists$ basis $B = \{b_1, \dots, b_n\}$ with the properties:
  - $[b_i, b_j] \subset B \cup B' \cup \{1\}$
  - For any 3 elements at least 2 commute
  - $a, b \in B$ then $[a b]$ commutes with both a & b

<u>maximal monomial</u>

Fact Simply laced Dynkin diagrams define maximal monomial groups.

Quest. Are all maximal monomial defined by Dynkin diagrams (simply laced)

Construction: Q quiver simply laced Dynkin
Φ_+ positive roots

Define G: $\begin{cases} \text{generators} & \theta(\alpha),\ \alpha \in \Phi_+ \\ \text{relations} & [\theta(\alpha)\theta(\beta)] = \begin{matrix} \theta(\alpha+\beta) \text{ if } \alpha+\beta \in \Phi_+ \\ 0 \quad \text{otherw.} \end{matrix} \end{cases}$

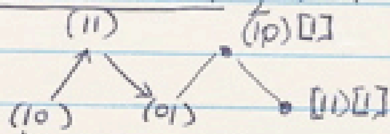
Work in progress – define G in general

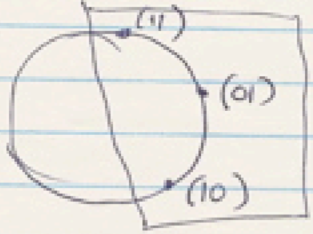
6

BACK TO PICTURES
"IGUSA-ORR" vs "CLUSTER-SEMIINVARIANT"

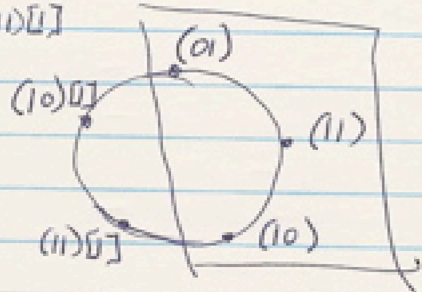
A_2

Auslander-Reiten quiver and cluster category





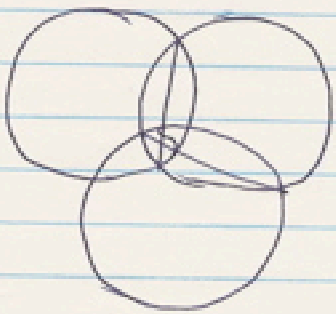
Igusa Orr

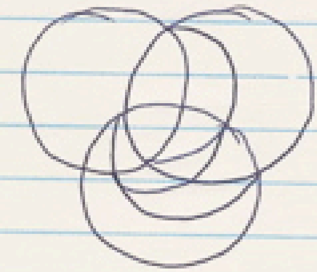


cluster picture

Recursion:

A_3





12 $\triangle$
2 $\square$
1 $\bigcirc$

14 TRIANGLES