Geometry 1, MTH G122.
Fall 2004. Professor Mikhail Shubin.

Textbook:

Foundations of Differentiable Manifolds and Lie groups, by Frank W. Warner. Springer-Verlag New York, Inc., 1983.

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Homework assignment no. 8
(due November 9)

1. Let $\delta : \Lambda^\bullet(M) \to \Lambda^\bullet(M)$ be an anti-derivation of the algebra of all exterior differential forms on a manifold $M = M^d$. Assume that δ has degree -1, i.e. maps $\Lambda^k(M)$ to $\Lambda^{k-1}(M)$ for all $k = 0, \ldots, d$. Prove that there exists a vector field X on M, such that $\delta = i(X)$.

2. Let $D : \Lambda^\bullet(M) \to \Lambda^\bullet(M)$ be a derivation of the algebra of all exterior differential forms on a manifold $M = M^d$. Assume that D has degree 0 (i.e. maps $\Lambda^k(M)$ to $\Lambda^k(M)$ for all $k = 0, \ldots, d$) and commutes with the exterior differentiation d. Prove that there exists a vector field X on M, such that D is the corresponding Lie derivative, i.e. $D = L_X$.