Geometry 1, MTH G122.
Fall 2004. Professor Mikhail Shubin.

Textbook:

Foundations of Differentiable Manifolds and Lie groups, by Frank W. Warner. Springer-Verlag New York, Inc., 1983.

Office: 460 Lake Hall. **Phone:** (617)373-5676 **E-mail:** shubin@neu.edu

Homework assignment no. 6

(due October 26)

1. Let V be a finite-dimensional vector space over $\mathbb{R}$, and V^* the dual space. Denote by $p : V^* \otimes V \to \mathbb{R}$ the linear map induced by the canonical pairing between V^* and V. Let us transfer p to $\mathrm{Hom}_{\mathbb{R}}(V,V)$ using the canonical isomorphism $\varphi : V^* \otimes V \to \mathrm{Hom}_{\mathbb{R}}(V,V)$, i.e. consider the map $p \circ \varphi^{-1} : \mathrm{Hom}_{\mathbb{R}}(V,V) \to \mathbb{R}$. Prove that this map coincides with the usual trace $\mathrm{tr} : \mathrm{Hom}_{\mathbb{R}}(V,V) \to \mathbb{R}$.

2. With V, V^* as above consider the exterior algebras $\Lambda(V)$ and $\Lambda(V^*)$ with the non-degenerate duality between them defined by the formula

$$(v_1^* \wedge \ldots \wedge v_k^*, v_1 \wedge \ldots \wedge v_k) = \det\left[(v_i^*(v_j))_{1\le i,j\le k}\right], \quad v_i^* \in V^*,\ v_j \in V,$$

and so inducing isomorphism $\Lambda(V^*) \simeq (\Lambda(V))^*$. Identifying $(\Lambda(V))^*$ with the space $A(V)$ of all alternating multilinear maps $\omega : V \times \ldots \times V \to \mathbb{R}$, describe the action of the interior multiplication $i(v)$ on $A(V)$, where $v \in V$.