

Geometry 1, MTH G122.
Fall 2004. Professor Mikhail Shubin.

Textbook:

Foundations of Differentiable Manifolds and Lie groups, by Frank W. Warner. Springer-Verlag New York, Inc., 1983.

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Homework assignment no. 3

(due September 30)

1. Let us consider the 2-torus $\mathbf{T}^2 = S^1 \times S^1$, where the circle S^1 is understood as the set of complex numbers z with $|z| = 1$. Let us define a map $\varphi : \mathbf{R} \rightarrow \mathbf{T}^2$ by setting $\varphi(t) = (e^{2\pi it}, e^{2\pi i\alpha t})$, where $\alpha \in \mathbf{R}$ is irrational. Prove that (\mathbf{R}, φ) is a dense submanifold in \mathbf{T}^2 . (It is known as the *skew line on the torus*.)

2. A *Riemannian structure* on a C^∞ manifold M is a smooth choice of a positive definite inner product $\langle \cdot, \cdot \rangle_p$ on each tangent space M_p , smooth in the following sense: if X and Y are C^∞ vector fields on M , then $\langle X, Y \rangle(p) = \langle X_p, Y_p \rangle$ is a C^∞ function of $p \in M$. Prove that a Riemannian structure exists on every C^∞ manifold M .

Hint: Use partition of unity on M .

Remark. A *Riemannian manifold* is a C^∞ manifold with a Riemannian structure.