Complex Analysis, MTH G204.
Fall 2004. Professor Mikhail Shubin.

Textbook:

Complex Analysis, by Theodore W. Gamelin. Springer-Verlag New York, Inc., 2001. Corrected printing 2003

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Homework assignment no. 5
(due October 14)

A general remark. Problems marked with * have an extended due date, which is not specified now. They are usually more difficult, but on the other hand, they are definitely worth thinking about. Please hand me any one of them when it is ready. Some of them may be topics for projects.

1. Section V.3, page 143-144: 1(c,d,h,i), 3(a), 4.

2. Section V.4, page 147-149: 1(a,b,c,d), 3, 7, 10.

3. Section V.5, page 151: 1(a, c).

4. Section V.6, page 153-154: 2, 4.

5. Section V.7, page 157-158: 1(a,g,e), 4, 5, 6, 7.

6. Section V.8, page 162-164: 2, 9.

7*. Calculate the limit of

$$\frac{\sin(\tan x) - \tan(\sin x)}{\sin^{-1}(\tan^{-1}(x)) - \tan^{-1}(\sin^{-1}(x))}$$

as $x \to 0$. (Here $\sin^{-1}$ and $\tan^{-1}$ are the principal branches, such that $\sin^{-1}(0) = \tan^{-1}(0) = 0$.)

A comment. It is possible in principle to use L'Hospital's rule (applied several times), but the calculations are extremely complicated (if done without computer's help), and mistakes are inavoidable on this path. But calculations with the help of power series decompositions are manageable (though still not very short), and this is the solution which I recommend you to try. There is also a very short and elegant geometric solution, but it seems very difficult to find.

V.I.Arnold quoted this problem in his book "Huygens and Barrow, Newton and Hooke. Pioneers in mathematical analysis and catastrophe theory from evolvents to quasicrystals." (Translated from the Russian by Eric J. F. Primrose. Birkhäuser Verlag, Basel, 1990.) He wrote: "This is an example of a problem which would be solved by people like Barrow, Newton, Huygens in a few minutes. But, as far as I know, modern mathematicians are unable to do it fast (at least I have never met a mathematician who could make it fast)."