Corrections for Second Edition

Partial Differential Equations: Methods and Applications
by Robert McOwen

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p. 115-116. The proof of (ii) of the Proposition contains a couple of typos: there should be a 2π in front of $\int_{|x|<2\epsilon} \epsilon^{-1} K(z)\, dz$ when $n = 2$, and $\int_{|x|<2\epsilon} \partial |K(z)/\partial z_j| dz$ should be $\int_{|x|<2\epsilon} |\partial K(z)/\partial z_j| dz$. Moreover, the proof of (iii) is incorrect. The correct proof is the following.

To prove (iii), let $v = \partial_j u = u_{(j)}$, then introduce

$$v_{(k)}(x) = \int_\Omega \partial_k \partial_j K(x-y)(f(y) - f(x))\, dy - f(x) \int_{\partial\Omega} \partial_j K(x-y) \nu_k \, dS_y.$$

(The domain integral converges because $f \in C^1$ implies $f(y) - f(x) = O(|y-x|)$ as $y \to x$.) We want to show that $v_{(k)} = \partial_k v$. Using the same function $\eta(t)$ as before, let us introduce the smooth function

$$v_\epsilon(x) = \int_\Omega \partial_j K(x-y)\, \eta\left(\frac{|x-y|}{\epsilon}\right) f(y)\, dy.$$

Clearly, $v_\epsilon \to v$ uniformly on compact subsets of $\mathbf{R}^n$ as $\epsilon \to 0$. For any fixed $x \in \Omega$, write

$$\partial_k v_\epsilon(x) = \int_\Omega \partial_k \left[\partial_j K(x-y)\, \eta\left(\frac{|x-y|}{\epsilon}\right)\right] (f(y) - f(x))\, dy + f(x) \int_\Omega \partial_k \left[\partial_j K(x-y)\, \eta\left(\frac{|x-y|}{\epsilon}\right)\right] dy.$$

We can take $2\epsilon < \mathrm{dist}(x, \partial\Omega)$ to find

$$\int_\Omega \partial_k \left[\partial_j K(x-y)\, \eta\left(\frac{|x-y|}{\epsilon}\right)\right] dy = -\int_{\partial\Omega} \partial_j K(x-y) \nu_k \, dS_y,$$

where $\vec{\nu}$ is the exterior unit normal to Ω. Thus

$$v_{(k)}(x) - \partial_k v_\epsilon(x) = \int_{|x-y|<2\epsilon} \partial_k \left[\left(1 - \eta\left(\frac{|x-y|}{\epsilon}\right)\right) \partial_j K(x-y)\right] (f(y) - f(x))\, dy.$$

But, arguing as in the previous paragraph, we can show that

$$\int_{|x-y|<2\epsilon} \partial_k \left[\left(1 - \eta\left(\frac{|x-y|}{\epsilon}\right)\right) \partial_j K(x-y)\right] |x-y|\, dy \le C(\epsilon),$$

where $C(\epsilon) \to 0$ as $\epsilon \to 0$, so

$$|v_{(k)}(x) - \partial_k v_\epsilon(x)| \le C(\epsilon) \sup_{y \in \Omega} \frac{|f(y) - f(x)|}{|y-x|}.$$

But $f \in C^1(\overline{\Omega})$ ensures that the supremum is finite and continuous in $x \in \Omega$, so we conclude that $\partial_k v_\epsilon \to v_{(k)}$ uniformly on compact neighborhoods of x as $\epsilon \to 0$. In particular, $v \in C^1(\Omega)$, and hence $u \in C^2(\Omega)$. ♠

p. 118. "In the next section" should be "In the next two subsections," and $C^2(\overline{\Omega})$ should be replaced by $C^2(\Omega) \cap C^1(\overline{\Omega})$ right after (33) and in the last paragraph of this subsection.

p. 155. In the last paragraph, replace "Exercise 6 in Section 2.3" by "the Lemma in Section 2.3".

p. 202. In the paragraph following Theorem 5, replace $\psi(U_i \cap B_r(\xi)) \subset \mathbf{R}^n_+$ by $\psi(\Omega \cap U_i) \subset \mathbf{R}^n_+$. In the first line of the proof of Lemma 1, $C^\infty(\Omega)$ should be $C^\infty(\overline{\Omega})$.

p. 208. Displayed formula (86) should be $G(T) = \{(x, Tx) : x \in D\} \subset X \times X$.

p. 210. The proof of Proposition 2 contains typos and is not very clear. First of all, $J^2 \neq I$ but rather $J^2 = -I$; however, this means $J^2M = M$ for any subspace M of $X \times X$. Secondly, J is a *unitary* isomorphism, i.e. $\langle Jv, Jm\rangle = \langle v, m\rangle$, so $v \in M^\perp \Leftrightarrow Jv \in (JM)^\perp$ and hence $J(M^\perp) = (JM)^\perp$. Finally, $G(T)$ need not be a closed subspace of $X \times X$, but by Exercise 12 in Section 6.1 we have $\overline{G(T)} = (G(T)^\perp)^\perp$.

p. 254. In the last paragraph of the proof of Theorem 1, it is asserted that $\Delta(\phi u) = f'$ where $f' \in H^k(\mathbf{T}^n)$; in fact, this should be $f' \in L^2(\mathbf{T}^n)$. This effects the next step of the proof, since Theorem 2 of Section 8.1 now only yields $\phi u \in H^2(\mathbf{T}^n)$, which in turn implies $u \in H^2(\Omega')$. In order to obtain $u \in H^{k+2}(\Omega')$, this argument may be iterated, but some additional care must be taken because we always need to shrink the domain whenever we increase the regularity. Thus, in the first step, we should replace Ω' by Ω_1 and replace ϕ by the cutoff function $\phi_1 \equiv 1$ on Ω_1 to conclude $u \in H^2(\Omega_1)$. Iterating this argument yields $u \in H^3(\Omega_2)$, $u \in H^4(\Omega_3)$, etc. The condition $f \in H^k(\Omega)$ allows this iteration to continue until $u \in H^{k+2}(\Omega_{k+1})$ is obtained, and this implies $u \in H^{k+2}(\Omega')$.

p. 256. The condition $0 < |h| < \mathrm{dist}(x, \partial\Omega)$ should be $0 < |h| < \mathrm{dist}(\mathrm{supp}\,\phi, \partial\Omega)$, and formula (35b) should be

$$\text{(35b)} \qquad \int_\Omega \phi\, \delta_j^{h_k} u\, dx = \int_\Omega (\delta_j^{-h_k}\phi)\, u\, dx \to -\int_\Omega \frac{\partial \phi}{\partial x_j} u\, dx.$$

p. 258. In the last sentence of Theorem 4, $u - \phi \in H_0^{1,2}(\Omega)$ should be $u - \phi \in H_0^1(\Omega)$, since we generally write $H_0^{1,2}(\Omega)$ as $H_0^1(\Omega)$ throughout this Chapter.

p. 266. In the second line from the bottom, $\Gamma \subset \Omega^+ \cap \Omega$ should read $\Gamma \subset \partial\Omega^+ \cap \Omega$.

p. 267. The first sentence of Exercise 1 is incorrect: $C = AB \geq 0$ is not necessarily true. The correct sentence reads as follows. Recall from linear algebra that if A and B are symmetric $n \times n$ matrices and $A, B \geq 0$, then the *trace* of $C = AB = (c_{ij})$ is nonnegative: $\mathrm{tr}(C) \equiv c_{11} + c_{22} + \cdots c_{nn} \geq 0$.

p. 284. In at least two places, "eigenvector λ" should obviously read "eigenvalue λ".

p. 395. The term "nonnegative" appears erroneously twice on this page: in the proof of Proposition 2 it should be simply deleted ("and a $u_0 \in X = H_0^{1,2}(\Omega)$"), and near the bottom of the page it should be replaced by "nontrivial" ("u_0 is a nontrivial weak solution").

p. 397. Remove $(\mathrm{vol}\,\Omega)^2$ from the inequality; in fact C_ϵ may easily be found to be $(4\epsilon)^{-1}$.

p. 414. Just below (11), k should be κ (but k in (12) is correct).

p. 425. In Section 3.1 #3 (a), the formula should have $\cos(kt)$ instead of $\sin(kt)$.

p. 427. In Section 4.1, #8, $\Delta v < 0$ should be $\Delta v > 0$.
In Section 4.2, #3, $B_\epsilon(x) \cap B_\epsilon(\xi)$ should be $B_\epsilon(x) \cup B_\epsilon(\xi)$ and $u(y) = G(x, y)$ should be $u(y) = G(y, x)$.

p. 428. In Section 4.3, #1, the last half of the sentence should read "...apply (31) in Ω' with the Green's function $G(x, \xi)$ for Ω'.

p. 433. The answer/hint should have been changed for the second edition! For the Neumann problem $\Delta u = 0$ in Ω, $\partial u/\partial \nu = g$ on $\partial\Omega$, the definition of a weak solution is $u \in H^{1,2}(\Omega)$ such that

$$\int_\Omega u \Delta v \, dx + \int_{\partial\Omega} (gv - u\partial v/\partial \nu) \, ds \quad \text{for all } v \in C^\infty(\overline{\Omega}).$$

Use $F(u) = \frac{1}{2}\int_\Omega |\nabla u|^2 \, dx - \int_{\partial\Omega} gu \, ds$ on the admissible set $\mathcal{A} = C^1(\overline{\Omega})$.

Note. *Thanks to Hal L. Smith of Arizona State University for bringing many of these errors to the author's attention.*