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★**Power sums, Gorenstein algebras, and determinantal loci.**

Appendix C by Iarrobino and Steven L. Kleiman.

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The book under review has been "in the works" for quite a few years, and an earlier manuscript has often been quoted in the literature. The final product is well worth the wait. It gives an extremely thorough and self-contained treatment of several very interesting, and technically demanding, areas of algebraic geometry, and it describes in detail the fascinating ways in which they are intertwined. Many results given in the book are new, and many more are quoted from other sources, usually with at least an idea of the proof. Indeed, a very valuable feature is the list of over 300 references.

From the preface: "This book is devoted to a classical problem with a long history—that of representing a homogeneous polynomial as a sum of powers of linear forms. This problem is closely related to another interesting topic—the study of the loci which parameterize homogeneous polynomials with a given sequence of dimensions for the spaces spanned by their order-i higher partial derivatives. Here a convenient tool to work with are the catalecticant matrices associated to a homogeneous polynomial, whose columns are the coefficients of its partial derivatives in appropriate monomial bases—the above dimensions are then the ranks of the catalecticant matrices, and the above parametric varieties are their determinantal loci."

Many of the notions covered in the book rely on the following set-up. Let $R = k[x_1, \cdots, x_r]$, let $x_i = \partial_i$ and view R as the ring of linear differential operators with constant coefficients acting on the ring $\mathcal{R} = k[X_1, \cdots, X_r]$ of polynomials. Let $f \in \mathcal{R}_j$, $f \neq 0$, and let $I \subset R$ be the ideal of all differential operators which annihilate f; we write $I = \mathrm{Ann}(f)$. It turns out that I is necessarily an Artinian Gorenstein ideal whose quotient ring has socle degree j.

We give a very rough idea of the content of the various chapters, but we stress that we cannot begin to do justice to the wealth of examples and remarks that are provided. The Introduction gives an "Informal History and Brief Outline" of the material in the book, which is very useful.

In Chapter 1 the authors give the preliminary material, examples and definitions involving determinantal schemes of catalecticant matrices, $\mathbf{V}_s(u, v; r)$, and they introduce the important object $\mathbf{Gor}(T)$,

with which much of the book deals. Very roughly, $\mathbf{Gor}(T)$ (or more precisely its associated reduced subscheme $\mathrm{Gor}(T)$) parameterizes the Artinian Gorenstein algebras with fixed Hilbert function T.

Chapter 2 begins with a discussion of the Waring problem, which deals with the minimum number of summands needed to represent a general form of degree j in r variables as a sum of powers of linear forms. It also gives an introduction to Artinian Gorenstein algebras, with the basic results and techniques needed in later chapters.

Chapter 3 reduces the calculation of the tangent spaces of $\mathbf{V}_s(u, v; r)$ and $\mathbf{Gor}(T)$ at a point f to determining the degree-j graded piece of the square I^2 of the Gorenstein ideal $I = \mathrm{Ann}(f)$ associated to f. It also provides some basic geometric tools to be used later.

Chapter 4 contains some of the main results of the book. The authors describe an irreducible component of $\mathrm{Gor}(T)$ for any r, and they show that when $r = 3$ this component is in fact all of $\mathrm{Gor}(T)$. They give more results for larger r.

Chapter 5 focuses on the case $r = 3$. After some technical preparation, the authors give their results on power sum representations, which involve all of the notions described earlier in the book. For example, if $f \in \mathrm{Gor}(T)$ and T has a subsequence (s, s, s), and f is general enough, then f has a length s power sum representation. If T does not have such a subsequence this is no longer true.

In Chapters 6, 7 and 8 the authors study the component structure of $\mathrm{Gor}(T)$ when $r \geq 4$, and its embeddings into projective space and into a product of Grassmannians. They also study the component structure of $\mathbf{PV}_s(u, v; r)$. Chapter 9 contains a discussion of open problems.

The authors also include a number of appendices containing background material, as well as an appendix written by the first author with S. Kleiman about Macaulay's growth conditions, and the connections to a number of articles, obtaining in the end an example of a scheme $\mathrm{Gor}(T)$ for $r = 4$ having several irreducible components.

Finally, the authors include a comparison of the current book with the earlier manuscript, since that manuscript has been quoted in a number of published articles, and they include a detailed index and an index of names mentioned in the book.

This book is obviously very carefully written, and a great effort was made by the authors to give as complete a description as possible (including large portions which are new) of an exciting and very active area. It is a valuable contribution to the field of algebraic geometry, and one which would be beneficial for experts and beginners alike.

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