

Probability Qualifying Exam 1-07.

① Let $X \sim \text{Bin}(n, \frac{1}{\sqrt{n}})$

Ⓐ Find the moment generating function of X

Ⓑ Show that $P(X \geq a\sqrt{n}) \leq \left[e^{-a(\log a + 1)} - 1 \right]^{\sqrt{n}}$
for $a > 1$. [Hint: $(1 + \frac{c}{\sqrt{n}})^{\sqrt{n}} \leq e^c$]

Ⓒ How do you know that $\frac{X - \sqrt{n}}{\sqrt{n}}$ is not approximately $N(0, 1)$?

② Let X_n be Geometric $(\frac{b}{n})$, that is,

$$P(X_n = x) = \left(1 - \frac{b}{n}\right)^{x-1} \left(\frac{b}{n}\right) \quad x = 1, 2, 3, \dots$$

Find the limiting distribution of $\frac{X_n}{n}$.

[Hint: you don't need mgf]