

# Probability Qualifying Jan. 2006

- ① An urn contains  $2n$  balls, 2 labelled ①, 2 labelled ②, ..., and 2 labelled ②.

$n$  balls are drawn without replacement.

Let  $M = \#$  of pairs drawn. Find  $EM$  and  $\text{var } M$

- ② Suppose  $N \sim \text{Poisson}(n)$ , i.e.  $P(N=j) = \frac{n^j}{j!} e^{-n}$

Let  $X_1, X_2, \dots$  be independent with

$$P(X_i=1) = P(X_i=-1) = P(X_i=0) = \frac{1}{3}$$

and let  $S_n = \sum_{j=1}^N X_j$ .

Ⓐ Find the characteristic function of  $S_n$ .

Ⓑ Find the limiting distribution of  $\frac{S_n}{\sqrt{n}}$ .

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$$P(X_i=1) = P(X_i=-1) = P(X_i=0) = \frac{1}{3}$$

Find a  $\lambda < 1$  such that

$$P(X_1 + \dots + X_n > \frac{n}{6}) \leq \lambda^n$$