

Prob. Qualifying

September 2006

① Draw 26 cards without replacement from an ordinary deck of 52 cards. (The deck contains 13 spades)

Let X = the number of spades obtained.

Find EX and $\text{var } X$.

② $X_n = Y_1 + Y_2 + \dots + Y_n + W_1 + W_2 + \dots + W_n$

where for all i , $P(Y_i = 1) = \frac{1}{3}$, $P(Y_i = 0) = \frac{2}{3}$

$P(W_i = 1) = \frac{2}{3}$, $P(W_i = 0) = \frac{1}{3}$

and all of $Y_1, Y_2, \dots, Y_n, W_1, W_2, \dots, W_n$ are independent

③ If $a > 1$, show that $\exists B < 1$ such that

$$P(X_n \geq an) \leq B^n \quad \forall n.$$

④ Find $\lim_{n \rightarrow \infty} P(X_n \geq n + \sqrt{n})$

⑤ Find $\lim_{n \rightarrow \infty} P(X_n \geq n + n^{1/4})$