

Northeastern University  
Mathematics Department

Qualifying Exam, General Algebra  
April 2010

- (1) Show that the ring  $\mathbb{Z}[\sqrt{-5}]$  is not a PID.
- (2) Is there an element  $d$  in  $\mathbb{Z}[\sqrt{-6}]$  such that the ring  $\mathbb{Z}[\sqrt{-6}]/(d)$  has 8 elements? If yes – give an example, if no – prove it.
- (3) How many units are there in the ring  $\mathbb{Z}[i]/(30)$ ?
- (4) Is  $1/8$  divisible by 10 in  $\mathbb{Z}(2^\infty)$ ? If yes – find the quotient, if no – explain why.